SOLUTION OF EXERCISE # 4.3

Exercise # 6.3

Q.1: Express each of the following sum or difference as products:

(i) $\sin 5\theta - \sin\theta$

(IA-2017), (IIA-2017)

Sol. $\sin 5\theta - \sin\theta$

$$= 2\cos\left(\frac{5\theta + \theta}{2}\right)\sin\left(\frac{5\theta - \theta}{2}\right)$$

$$= 2\cos\left(\frac{6\theta}{2}\right)\sin\left(\frac{4\theta}{2}\right)$$

$$= \boxed{2\cos 3\theta \sin 2\theta}$$

(ii) $\cos\theta - \cos 5\theta$

Sol. $\cos\theta - \cos 5\theta$

$$= -2\sin\left(\frac{\theta + 5\theta}{2}\right)\sin\left(\frac{\theta - 5\theta}{2}\right)$$

$$= -2\sin\left(\frac{6\theta}{2}\right)\sin\left(\frac{-4\theta}{2}\right)$$

$$= -2\sin 3\theta \sin(-2\theta)$$

$$= \boxed{2\sin 3\theta \sin 2\theta}$$

(iii) $\cos 12\theta + \cos 4\theta$

Sol. $\cos 12\theta + \cos 4\theta$

$$= 2\cos\left(\frac{12\theta + 4\theta}{2}\right)\cos\left(\frac{12\theta - 4\theta}{2}\right)$$

$$= 2\cos\left(\frac{16\theta}{2}\right)\cos\left(\frac{8\theta}{2}\right) = \boxed{2\cos 8\theta \cos 4\theta}$$

(iv) $\sin\frac{5\theta}{3} - \sin\frac{5\theta}{6}$

Sol. $\sin\frac{5\theta}{3} - \sin\frac{5\theta}{6} = 2\cos\left(\frac{\left(\frac{5\theta}{3} + \frac{5\theta}{6}\right)}{2}\right)\sin\left(\frac{\left(\frac{5\theta}{3} - \frac{5\theta}{6}\right)}{2}\right)$

$$= 2\cos\left(\frac{\left(\frac{10\theta + 5\theta}{6}\right)}{2}\right)\sin\left(\frac{\left(\frac{10\theta - 5\theta}{6}\right)}{2}\right) = \boxed{2\cos\left(\frac{15\theta}{12}\right)\sin\left(\frac{5\theta}{12}\right)}$$

SOLUTION OF EXERCISE # 4.3

(v) $\cos\left(\frac{\alpha+\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}\right)$

Sol. $\cos\left(\frac{\alpha+\beta}{2}\right)+\cos\left(\frac{\alpha-\beta}{2}\right)$

$$=2\cos\left(\frac{\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}}{2}\right)\cos\left(\frac{\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}}{2}\right)$$

$$=2\cos\left(\frac{\frac{\alpha+\beta+\alpha-\beta}{2}}{2}\right)\cos\left(\frac{\frac{\alpha+\beta-\alpha+\beta}{2}}{2}\right)$$

$$=2\cos\left(\frac{\frac{2\alpha}{2}}{2}\right)\cos\left(\frac{\frac{2\beta}{2}}{2}\right)=\boxed{2\cos\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right)}$$

(vi) $\sin 4\theta+\sin 2\theta$

Sol. $\sin 4\theta+\sin 2\theta$

$$=2\sin\left(\frac{4\theta+2\theta}{2}\right)\cos\left(\frac{4\theta-2\theta}{2}\right)$$

$$=2\sin\left(\frac{6\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)=\boxed{2\sin 3\theta\cos\theta}$$

Q.2: Express each of the following products as sum or difference:

(i) $2\sin 3\theta\cos\theta$ **(IIA-2019)**

Sol. $2\sin 3\theta\cos\theta$

$$=\sin(3\theta+\theta)+\sin(3\theta-\theta)=\boxed{\sin 4\theta+\sin 2\theta}$$

(ii) $\sin 3\theta\cos 5\theta$

Sol. $\sin 3\theta\cos 5\theta=\frac{1}{2}[2\sin 3\theta\cos 5\theta]$

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$$=\frac{1}{2}[\sin(3\theta+5\theta)+\sin(3\theta-5\theta)]$$

$$=\frac{1}{2}[\sin 8\theta+\sin(-2\theta)]=\boxed{\frac{1}{2}[\sin 8\theta-\sin 2\theta]}$$

(iii) $\cos 3\theta \cos 5\theta$

Sol. $\cos 3\theta\cos 5\theta=\frac{1}{2}(2\cos 3\theta\cos 5\theta)$

$$=\frac{1}{2}[\cos(3\theta+5\theta)+\cos(3\theta-5\theta)]$$

$$=\frac{1}{2}[\cos 8\theta+\cos(-2\theta)]=\boxed{\frac{1}{2}[\cos 8\theta+\cos 2\theta]}$$

(iv) $\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Sol. $=\frac{1}{2}\left[2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]$

$$=\frac{1}{2}\left[\sin\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right)+\sin\left(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}\right)\right]$$

$$=\frac{1}{2}\left[\sin\left(\frac{\alpha+\beta+\alpha-\beta}{2}\right)+\sin\left(\frac{\alpha+\beta-\alpha+\beta}{2}\right)\right]$$

$$=\frac{1}{2}\left[\sin\left(\frac{2\alpha}{2}\right)+\sin\left(\frac{2\beta}{2}\right)\right]=\boxed{\frac{1}{2}[\sin\alpha+\sin\beta]}$$

Q.3: **Express $\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta$ as a product.** (IIA-2019)

Sol. $\sin 3\theta+\sin 5\theta+\sin 7\theta+\sin 9\theta$

$$=2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right)+2\sin\left(\frac{7\theta+9\theta}{2}\right)\cos\left(\frac{7\theta-9\theta}{2}\right)$$

$$=2\sin\left(\frac{8\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)+2\sin\left(\frac{16\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)$$

SOLUTION OF EXERCISE # 4.3

$$= 2\sin 4\theta\cos(-\theta) + 2\sin 8\theta\cos(-\theta)$$

$$= 2\cos(-\theta)[\sin 4\theta + \sin 8\theta]$$

$$= 2\cos\theta\left[2\sin\left(\frac{4\theta+8\theta}{2}\right)\cos\left(\frac{4\theta-8\theta}{2}\right)\right] \because \cos(-\theta) = \cos\theta$$

$$= 2\cos\theta\left[2\sin\left(\frac{12\theta}{2}\right)\cos\left(\frac{-4\theta}{2}\right)\right]$$

$$= 4\cos\theta\sin 6\theta\cos(-2\theta) = \boxed{4\cos\theta\sin 6\theta\cos 2\theta}$$

Prove the following identities:

Q.4: $\dfrac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \tan\theta$

Sol. L.H.S. $= \dfrac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta}$

$$= \frac{2\cos\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right)}{2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}$$

$$= \frac{2\cos\left(\frac{8\theta}{2}\right)\sin\left(\frac{2\theta}{2}\right)}{2\cos\left(\frac{8\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)}$$

$$= \frac{2\cos 4\theta\sin\theta}{2\cos 4\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{R.H.S.}$$ **Proved.**

Q.5: $\dfrac{\sin 5\theta + \sin 3\theta}{\cos 5\theta - \cos 3\theta} = -\cot\theta$

Sol. L.H.S. $= \dfrac{\sin 5\theta + \sin 3\theta}{\cos 5\theta - \cos 3\theta}$

SOLUTION OF EXERCISE # 4.3

$$=\frac{2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}{-2\sin\left(\frac{5\theta+3\theta}{2}\right)\sin\left(\frac{5\theta-3\theta}{2}\right)}$$

$$=-\frac{\cos\left(\frac{2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}=-\frac{\cos\theta}{\sin\theta}=-\cot\theta \qquad = \text{R.H.S} \quad \textbf{Proved.}$$

Q.6: $\dfrac{\cos\beta+\cos 9\beta}{\sin\beta+\sin 9\beta}=\cot 5\beta$

Sol. $\text{L.H.S.}=\dfrac{\cos\beta+\cos 9\beta}{\sin\beta+\sin 9\beta}$

$$=\frac{2\cos\left(\frac{\beta+9\beta}{2}\right)\cos\left(\frac{\beta-9\beta}{2}\right)}{2\sin\left(\frac{\beta+9\beta}{2}\right)\cos\left(\frac{\beta-9\beta}{2}\right)}$$

$$=\frac{\cos\left(\frac{10\beta}{2}\right)}{\sin\left(\frac{10\beta}{2}\right)}$$

$$=\frac{\cos 5\beta}{\sin 5\beta}=\cot 5\beta=\text{R.H.S.} \qquad \textbf{Proved.}$$

Q.7: $\dfrac{\sin 3\theta-\sin\theta}{\cos^2\theta-\sin^2\theta}=2\sin\theta$

Sol. $\text{L.H.S.}=\dfrac{\sin 3\theta-\sin\theta}{\cos^2\theta-\sin^2\theta}$

$$=\frac{2\cos\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)}{\cos 2\theta} \quad \because \cos^2\theta-\sin^2\theta=\cos 2\theta$$

SOLUTION OF EXERCISE # 4.3

$$= \frac{2\cos 2\theta \sin\theta}{\cos 2\theta}$$

$$= 2\sin\theta = \text{R.H.S.}$$ **Proved.**

Q.8: $\dfrac{\sin A + \sin B}{\cos A + \cos B} = \tan\dfrac{A+B}{2}$

Sol. $\text{L.H.S.} = \dfrac{\sin A + \sin B}{\cos A + \cos B}$

$$= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \tan\left(\frac{A+B}{2}\right) = \text{R.H.S.}$$ **Proved.**

Q.9: $\dfrac{\cos 2\theta - \cos 6\theta}{\cos 2\theta + \cos 6\theta} = \tan 4\theta \tan 2\theta$

Sol. $\text{L.H.S.} = \dfrac{\cos 2\theta - \cos 6\theta}{\cos 2\theta + \cos 6\theta}$

$$= \frac{-2\sin\left(\frac{2\theta+6\theta}{2}\right)\sin\left(\frac{2\theta-6\theta}{2}\right)}{2\cos\left(\frac{2\theta+6\theta}{2}\right)\cos\left(\frac{2\theta-6\theta}{2}\right)}$$

$$= \frac{-\sin\left(\frac{8\theta}{2}\right)\sin\left(\frac{-4\theta}{2}\right)}{\cos\left(\frac{8\theta}{2}\right)\cos\left(\frac{-4\theta}{2}\right)}$$

$$= \frac{-\sin 4\theta \sin(-2\theta)}{\cos 4\theta \cos(-2\theta)}$$

SOLUTION OF EXERCISE # 4.3

$$= \frac{\sin 4\theta \sin 2\theta}{\cos 4\theta \cos 2\theta} \qquad \because \begin{cases} \sin(-2\theta) = -\sin 2\theta \\ \cos(-2\theta) = \cos 2\theta \end{cases}$$

$$= \tan 4\theta . \tan 2\theta = \text{R.H.S.}$$ **Proved.**

Q.10: $$\frac{\cos\alpha - \cos\beta}{\cos\alpha + \cos\beta} = -\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\cot\left(\frac{\alpha-\beta}{2}\right)}$$

Sol. $$\text{L.H.S.} = \frac{\cos\alpha - \cos\beta}{\cos\alpha + \cos\beta}$$

$$= \frac{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$= -\tan\left(\frac{\alpha+\beta}{2}\right)\tan\left(\frac{\alpha-\beta}{2}\right)$$

$$= -\frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\cot\left(\frac{\alpha-\beta}{2}\right)} = \text{R.H.S.}$$ **Proved.**

Q.11: $$\frac{\sin\theta + \sin 2\theta + \sin 3\theta}{\cos\theta + \cos 2\theta + \cos 3\theta} = \tan 2\theta$$

Sol. $$\text{L.H.S.} = \frac{\sin\theta + \sin 2\theta + \sin 3\theta}{\cos\theta + \cos 2\theta + \cos 3\theta}$$

$$= \frac{\sin 3\theta + \sin\theta + \sin 2\theta}{\cos 3\theta + \cos\theta + \cos 2\theta}$$

$$= \frac{2\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) + \sin 2\theta}{2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) + \cos 2\theta}$$

SOLUTION OF EXERCISE # 4.3

$$=\frac{2\sin\left(\frac{4\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)+\sin 2\theta}{2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)+\cos 2\theta}$$

$$=\frac{2\sin 2\theta\cos\theta+\sin 2\theta}{2\cos 2\theta\cos\theta+\cos 2\theta}$$

$$=\frac{\sin 2\theta(2\cos\theta+1)}{\cos 2\theta(2\cos\theta+1)}$$

$$=\frac{\sin 2\theta}{\cos 2\theta}=\tan 2\theta=\text{R.H.S.}$$ **Proved**

Q.12: $\sin 5\theta+2\sin 3\theta+\sin\theta=4\sin 3\theta\cos^2\theta$ **(IIA-2016)**

Sol. $\text{L.H.S.}=\sin 5\theta+2\sin 3\theta+\sin\theta$

$=\sin 5\theta+\sin\theta+2\sin 3\theta$

$$=2\sin\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right)+2\sin 3\theta$$

$$=2\sin\left(\frac{6\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)+2\sin 3\theta$$

$=2\sin 3\theta\cos 2\theta+2\sin 3\theta$

$=2\sin 3\theta(\cos 2\theta+1)$

$=2\sin 3\theta(2\cos^2\theta-1+1)$ $\quad\because \cos 2\theta=2\cos^2\theta-1$

$=2\sin 3\theta(2\cos^2\theta)$

$=4\sin 3\theta\cos^2\theta$ $\quad=\text{R.H.S.}$ **Proved.**

Q.13: Show that

(i) $$\frac{\sin 75^\circ-\sin 15^\circ}{\cos 75^\circ+\cos 15^\circ}=\frac{1}{\sqrt{3}}$$ **(IIA-2017)**

Sol. $$\text{L.H.S.}=\frac{\sin 75^\circ-\sin 15^\circ}{\cos 75^\circ+\cos 15^\circ}$$

SOLUTION OF EXERCISE # 4.3

$$= \frac{2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\sin\left(\frac{75^\circ-15^\circ}{2}\right)}{2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\cos\left(\frac{75^\circ-15^\circ}{2}\right)}$$

$$= \frac{\cancel{2}\cancel{\cos\left(\frac{90^\circ}{2}\right)}\sin\left(\frac{60^\circ}{2}\right)}{\cancel{2}\cancel{\cos\left(\frac{90^\circ}{2}\right)}\cos\left(\frac{60^\circ}{2}\right)}$$

$$= \frac{\sin 30^\circ}{\cos 30^\circ} = \tan 30^\circ = \frac{1}{\sqrt{3}} = \text{R.H.S.}$$ **Proved.**

(ii) $\sin 20^\circ + \sin 40^\circ = \cos 10^\circ$

Sol. L.H.S. $= \sin 20^\circ + \sin 40^\circ$

$$= 2\sin\left(\frac{20^\circ+40^\circ}{2}\right)\cos\left(\frac{20^\circ-40^\circ}{2}\right)$$

$$= 2\sin\left(\frac{60^\circ}{2}\right)\cos\left(\frac{-20^\circ}{2}\right)$$

$$= 2\sin 30^\circ\cos(-10^\circ) \qquad \because \cos(-\theta) = \cos\theta$$

$$= 2\left(\frac{1}{2}\right)\cos 10^\circ = \cos 10^\circ = \text{R.H.S.}$$ **Proved.**

(iii) $\cos 80^\circ + \cos 40^\circ = \cos 20^\circ$

Sol. L.H.S. $= \cos 80^\circ + \cos 40^\circ$

$$= 2\cos\left(\frac{80^\circ+40^\circ}{2}\right)\cos\left(\frac{80^\circ-40^\circ}{2}\right)$$

$$= 2\cos\left(\frac{120^\circ}{2}\right)\cos\left(\frac{40^\circ}{2}\right)$$

$$= 2\cos 60^\circ\cos 20^\circ$$

$$= 2\left(\frac{1}{2}\right)\cos 20^\circ = \cos 20^\circ = \text{R.H.S.}$$ **Proved.**

SOLUTION OF EXERCISE # 4.3

(iv) $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$ (IA-2016)

Sol. L.H.S. $= \cos 20^\circ + \cos 100^\circ + \cos 140^\circ$

$$= 2\cos\left(\frac{20^\circ + 100^\circ}{2}\right)\cos\left(\frac{20^\circ - 100^\circ}{2}\right) + \cos 140^\circ$$

$$= 2\cos\left(\frac{120^\circ}{2}\right)\cos\left(\frac{-80^\circ}{2}\right) + \cos 140^\circ$$

$$= 2\cos 60^\circ \cos(-40^\circ) + \cos 140^\circ$$

$$= 2\left(\frac{1}{2}\right)\cos 40^\circ + \cos 140^\circ \qquad \because \cos(-\theta) = \cos\theta$$

$$= \cos 40^\circ + \cos 140^\circ$$

$$= 2\cos\left(\frac{40^\circ + 140^\circ}{2}\right)\cos\left(\frac{40^\circ - 140^\circ}{2}\right)$$

$$= 2\cos\left(\frac{180^\circ}{2}\right)\cos\left(\frac{-100^\circ}{2}\right)$$

$$= 2\cos 90^\circ \cos(-50^\circ)$$

$$= 2(0)\cos 50^\circ = 0 = \text{R.H.S.}$$ **Proved.**

Q.14: $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$ (IA-2019)

Sol. L.H.S. $= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$$= \sin 20^\circ \sin 40^\circ \left(\frac{\sqrt{3}}{2}\right)\sin 80^\circ \qquad \because \left\{\sin 60^\circ = \frac{\sqrt{3}}{2}\right\}$$

$$= \frac{\sqrt{3}}{2}\left[\sin 20^\circ \sin 40^\circ \sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{2}\left[\frac{1}{-2}(-2\sin 20^\circ \sin 40^\circ)\sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{-4}\left[\left(\cos(20^\circ + 40^\circ) - \cos(20^\circ - 40^\circ)\right)\sin 80^\circ\right]$$

SOLUTION OF EXERCISE # 4.3

$$=\frac{\sqrt{3}}{-4}\left[\left(\cos 60^\circ-\cos(-20^\circ)\right)\sin 80^\circ\right]$$

$$=\frac{\sqrt{3}}{-4}\left[\left(\frac{1}{2}-\cos 20^\circ\right)\sin 80^\circ\right] \qquad \because \left\{\cos 60^\circ=\frac{1}{2}\right\}$$

$$=\frac{\sqrt{3}}{-4}\left[\left(\frac{1-2\cos 20^\circ}{2}\right)\sin 80^\circ\right]$$

$$=\frac{\sqrt{3}}{-8}[\sin 80^\circ-2\cos 20^\circ\sin 80^\circ]$$

$$=\frac{-\sqrt{3}}{8}[\sin 80^\circ-(\sin(20^\circ+80^\circ)-\sin(20^\circ-80^\circ))]$$

$$=\frac{-\sqrt{3}}{8}[\sin 80^\circ-\sin 100^\circ+\sin(-60^\circ)]$$

$$=\frac{-\sqrt{3}}{8}\left[2\cos\left(\frac{80^\circ+100^\circ}{2}\right)\sin\left(\frac{80^\circ-100^\circ}{2}\right)-\sin 60^\circ\right]$$

$$=\frac{-\sqrt{3}}{8}\left[2\cos\left(\frac{180^\circ}{2}\right)\sin\left(\frac{-20^\circ}{2}\right)-\frac{\sqrt{3}}{2}\right] \because \left\{\sin 60^\circ=\frac{\sqrt{3}}{2}\right\}$$

$$=\frac{-\sqrt{3}}{8}\left[2\cos 90^\circ\sin(-10^\circ)-\frac{\sqrt{3}}{2}\right]$$

$$=\frac{-\sqrt{3}}{8}\left[2(0)\sin(-10^\circ)-\frac{\sqrt{3}}{2}\right] \qquad \because \{\cos 90^\circ=0\}$$

$$=\frac{-\sqrt{3}}{8}\left(0-\frac{\sqrt{3}}{2}\right)=\frac{-\sqrt{3}}{8}\left(-\frac{\sqrt{3}}{2}\right)=\frac{3}{16}=\text{R.H.S.}$$ **Proved.**

Q.15: $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ=\dfrac{1}{16}$

Sol. L.H.S. $=\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ$

SOLUTION OF EXERCISE # 4.3

$$=\sin 10^\circ\left(\frac{1}{2}\right)\sin 50^\circ \sin 70^\circ \qquad \because \left\{\sin 30^\circ = \frac{1}{2}\right\}$$

$$=\frac{1}{2}\left[\sin 10^\circ \sin 50^\circ \sin 70^\circ\right]$$

$$=\frac{1}{2}\left[\frac{1}{-2}(-2\sin 10^\circ \sin 50^\circ)\sin 70^\circ\right]$$

$$=-\frac{1}{4}\left[(\cos(10^\circ+50^\circ)-\cos(10^\circ-50^\circ))\sin 70^\circ\right]$$

$$=-\frac{1}{4}\left[(\cos 60^\circ-\cos(-40^\circ))\sin 70^\circ\right]$$

$$=-\frac{1}{4}\left[\left(\frac{1}{2}-\cos 40^\circ\right)\sin 70^\circ\right]$$

$$=-\frac{1}{4}\left[\left(\frac{1-2\cos 40^\circ}{2}\right)\sin 70^\circ\right]$$

$$=-\frac{1}{8}\left[\sin 70^\circ-2\cos 40^\circ \sin 70^\circ\right]$$

$$=-\frac{1}{8}\left[\sin 70^\circ-(\sin(40^\circ+70^\circ)-\sin(40^\circ-70^\circ))\right]$$

$$=-\frac{1}{8}\left[\sin 70^\circ-\sin 110^\circ+\sin(-30^\circ)\right]$$

$$=-\frac{1}{8}\left[2\cos\left(\frac{70^\circ+110^\circ}{2}\right)\sin\left(\frac{70^\circ-110^\circ}{2}\right)-\frac{1}{2}\right]$$

$$=-\frac{1}{8}\left[2\cos 90^\circ \sin(-20^\circ)-\frac{1}{2}\right]$$

$$=-\frac{1}{8}\left[2(0)\sin(-20^\circ)-\frac{1}{2}\right]$$

$$=-\frac{1}{8}\left[0-\frac{1}{2}\right]=\frac{1}{16}=\text{R.H.S.}$$

Proved.

SOLUTION OF EXERCISE # 4.3

Q.16: $\sin 20^\circ \sin 40^\circ \sin 80^\circ \sin 90^\circ = \dfrac{\sqrt{3}}{8}$

Sol. L.H.S. $= \sin 20^\circ \sin 40^\circ \sin 80^\circ \sin 90^\circ$

$$= \sin 20^\circ \sin 40^\circ \sin 80^\circ (1) \qquad \because \{\sin 90^\circ = 1\}$$

$$= \frac{1}{-2}(-2\sin 20^\circ \sin 40^\circ)\sin 80^\circ$$

$$= \frac{1}{-2}\left[(\cos(20^\circ+40^\circ)-\cos(20^\circ-40^\circ))\sin 80^\circ\right]$$

$$= \frac{1}{-2}\left[(\cos 60^\circ - \cos(-20^\circ))\sin 80^\circ\right]$$

$$= \frac{1}{-2}\left[\left(\frac{1}{2}-\cos 20^\circ\right)\sin 80^\circ\right]$$

$$= \frac{1}{-2}\left[\left(\frac{1-2\cos 20^\circ}{2}\right)\sin 80^\circ\right]$$

$$= -\frac{1}{4}\left[\sin 80^\circ - 2\cos 20^\circ \sin 80^\circ\right]$$

$$= -\frac{1}{4}\left[\sin 80^\circ - \{\sin(20^\circ+80^\circ)-\sin(20^\circ-80^\circ)\}\right]$$

$$= -\frac{1}{4}\left[\sin 80^\circ - \sin(100^\circ) + \sin(-60^\circ)\right]$$

$$= -\frac{1}{4}\left[2\cos\left(\frac{80^\circ+100^\circ}{2}\right)\sin\left(\frac{80^\circ-100^\circ}{2}\right) - \frac{\sqrt{3}}{2}\right]$$

$$= -\frac{1}{4}\left[2\cos 90^\circ \sin(-10^\circ) - \frac{\sqrt{3}}{2}\right]$$

$$= -\frac{1}{4}\left[2(0)\sin(-10^\circ) - \frac{\sqrt{3}}{2}\right] \qquad \because \{\cos 90^\circ = 0\}$$

$$= -\frac{1}{4}\left[0 - \frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}}{8} = \text{R.H.S.}$$ **Proved.**

SOLUTION OF EXERCISE # 4.3

Q.17: $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \dfrac{1}{16}$ **(IA-2018)**

Sol. $\text{L.H.S.} = \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$$= \cos 20^\circ \cos 40^\circ \left(\frac{1}{2}\right) \cos 80^\circ \qquad \because \left\{\cos 60^\circ = \frac{1}{2}\right\}$$

$$= \frac{1}{2}\left[\cos 20^\circ \cos 40^\circ \cos 80^\circ\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ\right]$$

$$= \frac{1}{4}\left[\left(\cos(20^\circ+40^\circ) + \cos(20^\circ-40^\circ)\right)\cos 80^\circ\right]$$

$$= \frac{1}{4}\left[\left(\cos 60^\circ + \cos(-20^\circ)\right)\cos 80^\circ\right]$$

$$= \frac{1}{4}\left[\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ\right] = \frac{1}{4}\left[\left(\frac{1+2\cos 20^\circ}{2}\right)\cos 80^\circ\right]$$

$$= \frac{1}{8}\left[\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ\right]$$

$$= \frac{1}{8}\left[\cos 80^\circ + \cos(20^\circ+80^\circ) + \cos(20^\circ-80^\circ)\right]$$

$$= \frac{1}{8}\left[\cos 80^\circ + \cos 100^\circ + \cos(-60^\circ)\right]$$

$$= \frac{1}{8}\left[2\cos\left(\frac{80^\circ+100^\circ}{2}\right)\cos\left(\frac{80^\circ-100^\circ}{2}\right) + \cos 60^\circ\right]$$

$$= \frac{1}{8}\left[2\cos 90^\circ \cos(-10^\circ) + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[2(0)\cos 10^\circ + \frac{1}{2}\right] \qquad \because \left\{\begin{array}{l}\text{Using calculator} \\ \cos 90^\circ = 0\end{array}\right\}$$

$$= \frac{1}{8}\left[0 + \frac{1}{2}\right] = \frac{1}{16} = \text{R.H.S.}$$

Proved.